

NAG Toolbox for MATLAB

c02al

1 Purpose

c02al determines the roots of a quartic equation with real coefficients.

2 Syntax

```
[zeror, zeroi, errest, ifail] = c02al(e, a, b, c, d)
```

3 Description

c02al attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e, a, b, c and d are real coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

The eigenvalues are obtained by a call to f08pe. Further details can be found in Section 8.

To obtain the roots of a cubic equation, c02ak can be used.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **e – double scalar**

e , the coefficient of z^4 .

Constraint: $e \neq 0.0$.

2: **a – double scalar**

a , the coefficient of z^3 .

3: **b – double scalar**

b , the coefficient of z^2 .

4: **c – double scalar**

c , the coefficient of z .

5: **d – double scalar**

d , the constant coefficient.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **zeror(4)** – double array

2: **zeroi(4)** – double array

zeror(i) and **zeroi**(i) contain the real and imaginary parts, respectively, of the i th root.

3: **errest(4)** – double array

errest(i) contains an approximate error estimate for the i th root.

4: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **e** = 0.0.

ifail = 2

The companion matrix H cannot be formed without overflow.

ifail = 3

The iterative procedure used to determine the eigenvalues has failed to converge.

7 Accuracy

If **ifail** = 0 on exit, then the i th computed root should have approximately $|\log_{10}(\mathbf{errest}(i))|$ correct significant digits.

8 Further Comments

The method used by the function consists of the following steps, which are performed by routines from LAPACK in Chapter F08.

- (a) Form matrix H .
- (b) Apply a diagonal similarity transformation to H (to give H').
- (c) Calculate the eigenvalues and Schur factorization of H' .
- (d) Calculate the left and right eigenvectors of H' .
- (e) Estimate reciprocal condition numbers for all the eigenvalues of H' .
- (f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

9 Example

```
e = 1;
a = 2;
b = 6;
c = -8;
d = -40;
[zeror, zeroi, errest, ifail] = c02al(e, a, b, c, d)

zeror =
    2.0000
   -2.0000
   -1.0000
   -1.0000
zeroi =
     0
     0
    3.0000
   -3.0000
errest =
    1.0e-14 *
    0.0891
    0.1101
    0.1003
    0.1003
ifail =
         0
```
